CORROSION MONITORING OF SACRIFICIAL ANODES BASED ON CONTOUR PLOT ANALYSIS OF ELECTRO-MECHANICAL IMPEDANCE SPECTRA

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ABSTRACT
In this paper, we utilize a graphical technique originally proposed for damage assessment in beams, to determine the extent of corrosion of sacrificial zinc anode discs instrumented with piezoelectric transducers using Electro-Mechanical Impedance (EMI) measurements. The underlying parameters determining extent of corrosion (thicknesses of the zinc and zinc oxide films) are extracted from measured resonance frequencies of radial and transverse vibration modes by finding roots of the characteristic equations of these modes graphically through contour plot. This technique shows excellent agreement with experimental results, requires no calibration and is computationally inexpensive unlike optimization techniques.

KEYWORDS
Structural health monitoring, contour plot technique, corrosion monitoring, resonant smart sensor

INTRODUCTION
Vibration characteristics of a structure, namely frequency, mode shape and damping characteristics, are affected by the physical characteristics of a structure, and are used as sensing parameters in structural health monitoring (SHM). Several techniques have been reported for the damage localization (determining the geometric location of the damage) as well as damage quantification (evaluation of the severity of the damage) [1]. Natural frequency-based techniques require a single point measurement and shows greater accuracy, ease and reliability over the other vibration-based techniques [2,3]. The 'inverse problem' in the field of SHM, quantifies the damage in a structure (i.e. change in geometr) through changes in natural frequencies, by solving simultaneous non-linear equations [4]. Analytical, Finite Element Analysis (FEA) and optimization-based inverse techniques have been reported showing the effectiveness of frequency based graphical technique (using contour plots) in detection of delamination and cracks in composite/isotropic beams. Contour plot technique for analyzing shift in resonance frequencies is highly effective for estimation of damage location and quantification of extent of damage, without requiring computationally expensive iterations or network training [4-8].

Corrosion affects the dynamic properties of a structure, thereby causing shift in resonance frequencies. In previous work on this subject, Tamhane et al. presented an analytical model for measuring the extent of corrosion by monitoring the shift in resonance radial in-plane frequency obtained through Electro-Mechanical Impedance (EMI) measurements [9]. In this work we develop an alternate technique to estimate the extent of corrosion of sacrificial anodes by utilizing the contour plot technique applied to three modes of vibration of the disc; namely the radial expansion mode and two fundamental out-of-plane (transverse or `trampoline') vibration modes of the disc. The contour plot technique does not require a model of delamination dynamics of corrosion byproducts and therefore requires no calibration. The technique is validated through extensive FEA and extended to resonance frequency values obtained from experimental EMI measurements conducted with an impressed-current based accelerated corrosion system. The contour plot technique is thus suitable for in-situ monitoring of sacrificial anodes used in cathodic protection systems.

MATHEMATICAL FORMULATION
This section provides analytical models for determining the resonance frequencies of the radial in-plane axisymmetric and transverse modes of vibration of a circular plate. A photograph of the smart anode (zinc anode instrumented with PZT transducer) is shown in Figure 1(a).

Axial In-plane Vibration of Circular Plates
The model proposed by Tamhane et al. [9] represents the corroded metal disc with PZT transducer as a composite structure comprised of two portions as shown in Figure 1(b). The general solutions for the individual portions are obtained by extending the constituent equations for vibrations of a circular plate [10].

For a twin layered circular disc with equivalent circular PZT transducer undergoing axisymmetric in-plane radial vibration, the following boundary conditions apply: (i) free edge boundary condition at $r = a$ is $N_1(r) = 0$, i.e. $\left.\frac{\partial U_1}{\partial r}\right|_{r=a} + \nu_{1eq}\left.\frac{U_1}{r}\right|_{r=a} = 0$, and (ii) continuity of displacement and radial forces at the interface between portion I and portion II (at $r = b$). Accounting for these boundary conditions in the solutions for displacement, a 3×3 characteristic determinant Δ_R is obtained [9]. Since the material properties (Young's modulus E, Poisson's ratio ν and density ρ) for the three layers (metal anode, metal oxide and the PZT transducer) are known, the characteristic equation $|\Delta_R|$ is reduced to a non-linear function in natural frequency (ω_r), thickness of corroded metal oxide layer (h_1) and thickness of metal anode layer (h_2).

Transverse Vibration of Circular Plates
The general solutions for transverse displacement and shear angles for individual portions are obtained from the constitutive equations of a thick circular plate undergoing transverse vibration [11]. For a bi-layer circular thick plate with equivalent circular transducer undergoing free transverse vibrations, the free edge boundary conditions at $r = a$ are zero moment, $M_r = \frac{\partial \phi_r}{\partial r} + \frac{\nu}{r}\left(\phi_r + \frac{\partial \phi_\theta}{\partial \theta}\right) = 0$,

$M_{r\theta} = \frac{\partial \phi_\theta}{\partial r} + \frac{1}{r}\left(\frac{\partial \phi_r}{\partial \theta} - \phi_\theta\right) = 0$ and zero shear force, $Q_r = \frac{\partial w_1}{\partial r} + \phi_r = 0$. At the interface between portion I and portion II, there exists continuity of transverse displacement, shear angles (ϕ_r, ϕ_θ), moments and shear force. Substituting the solutions for displacement and shear angles, we obtain a 9×9 characteristic determinant $|\Delta_T|$. With prior knowledge of material properties (E, ν and ρ) for the three layers, the characteristic determinant $|\Delta_T|$ can be expressed as a function of the natural resonance frequency of transverse vibration, ω_t, and thickness of the corroded (zinc oxide) layer h_1, and the uncorroded (zinc) layer h_2. Equating the characteristic equations for all modes (radial and transverse) to zero, the values of thicknesses h_1 and h_2 can be determined by measuring the natural frequencies and solving the system of simultaneous non-linear equations for all modes.

CONTOUR PLOT TECHNIQUE

For any mode of vibration, a three-dimensional surface can be generated with solution of the characteristic equation displayed on the z-axis and h_1 and h_2 in the two orthogonal directions in the horizontal plane. Intersection of this plot with horizontal plane at $z = 0$ (i.e. projection on the $h_1 - h_2$ plane) yields all combinations of h_1 and h_2 that are solutions of the characteristic equation for the measured natural frequency. The '*contour*' plot function of MATLAB® is used for plotting these curves at $z = 0$. A minimum of such two curves are required to obtain a unique solution for h_1 and h_2. The coordinates of the intersection point of these 2D projections in the x-y plane correspond to the solutions for h_1 and h_2. While two modes are sufficient, considering that we are solving for two unknowns, it is preferred to use at least three modes to avoid cases where two curves intersect at multiple points. The common intersection of the three curves of any three fundamental natural frequencies gives the thicknesses (Figures 2 and 3). If there is no unique intersection point of three curves, centroid of the triangle formed by three-paired intersection is taken as the solution for h_1 and h_2.

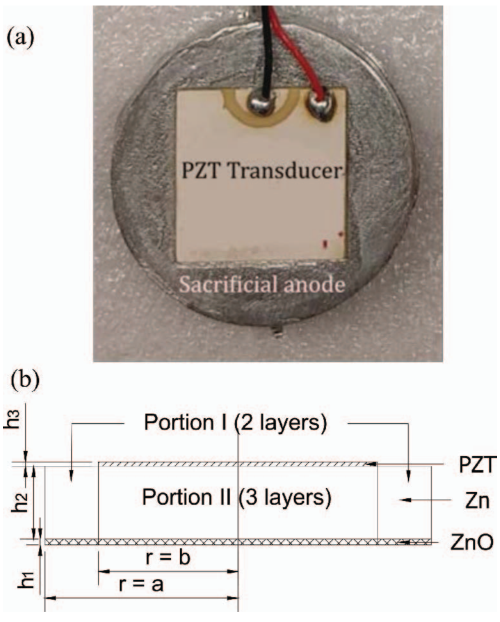

Figure 1: (a) Photograph of PZT transducer bonded to zinc sacrificial anode. (b) Cross-sectional view of the circular anode with equivalent circular PZT transducer on top and uniformly formed oxide layer on bottom surface.

RESULTS AND DISCUSSIONS

Finite Element Analysis (FEA)

The accuracy of the proposed contour plot technique is validated through FEA simulations using COMSOL Multiphysics. The simulations are performed using 'Normal' mesh setting in COMSOL Multiphysics, considering trade-off between computation time and convergence. The resonance frequencies for the fundamental in-plane axisymmetric radial mode and two fundamental transverse modes (mode shapes shown in Figure 2(a), (b) and (c) respectively), are obtained from the combination of h_1 and h_2 programmed into the FE software. The resonance frequencies are then used to estimate h_1 and h_2 using the contour plot technique, that are then compared to the programmed values. Here $\mu_{cor} = h_1/_{h_1 + h_2}$ and $\mu_{uncor} = h_2/_{h_1 + h_2}$ and represent the normalized thicknesses of the corroded and uncorroded regions of the anode respectively. The model chosen for the analysis (Figure 1(a)) is based on a commercially available

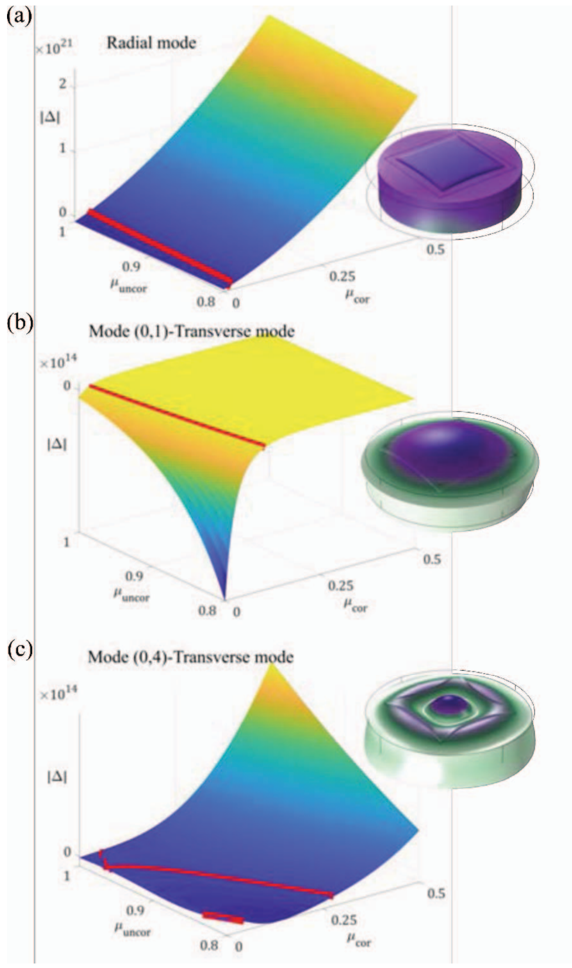

Figure 2: Example surface plots of characteristic equation for vibration modes against corrosion thickness ratios and intersections of horizontal plate at $|\Delta| = 0$ (in red) for programmed $\mu_{uncor} = 0.980$ and $\mu_{cor} = 0.031$.

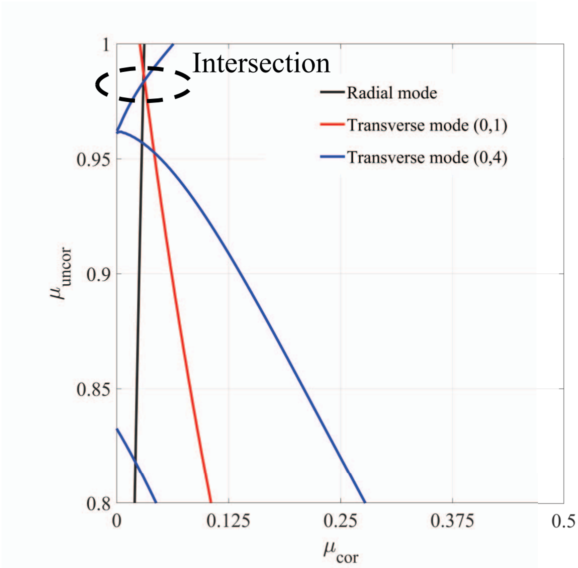

Figure 3: Example 2-D plots of characteristic equations for programmed μ_{uncor} = 0.980 and μ_{cor} = 0.031.

cylindrical zinc sacrificial anode (Krishna Conchem Products Pvt. Ltd.) of diameter 36 mm and thickness 7.3mm and PZT-5H transducer of 20mm side square and 0.4mm thickness (SP-5H, Sparkler Ceramics Pvt. Ltd.). The equivalent intersection of the three 2D contour plots are shown in Figure 3, corresponding to programmed values: $\mu_{uncor} = 0.980$ and $\mu_{cor} = 0.031$. The estimated values using contour plot technique are $\mu_{uncor} = 0.985$ and $\mu_{cor} = 0.030$.

Experimental Results: Corrosion monitoring of sacrificial zinc anodes

Accelerated corrosion experiments were performed on two separate commercially available zinc sacrificial anodes instrumented with PZT transducer (sample A and B). The PZT transducer is attached using Fevikwik® instant adhesive at the center of one of the flat surfaces on the circular zinc anode surface. To ensure water proofing, PZT transducer is coated with an epoxy sealant (M-Seal® Clear RTV Silicone Sealant cured for 12h). The anodes are subjected to impressed current of 0.35A for varying durations of time using the experimental setup detailed in [9]. The measured resonance frequencies for the fundamental radial mode and two transverse modes: (0,1) & (0,4) are recorded every 30 minutes and the shifts in resonance frequencies are noted. The values of these frequencies are programmed as inputs to the contour plot algorithm to estimate the thicknesses of ZnO layer (h_1) and Zn layer (h_2). The results for samples A and B are tabulated in Tables 1 and 2 respectively. To evaluate the accuracy of the proposed model, the estimated thicknesses are plugged into COMSOL Multiphysics to compare the frequency shift of the above-mentioned modes obtained through FEA vs. experiment. Figure 4 shows excellent agreement in the trend for all three eigenfrequencies obtained from COMSOL with those measured experimentally.

Table 1: Thickness of Zn (h_2) and ZnO (h_1) layers estimated from resonance frequencies of Sample A.

Time (min)	Radial mode (kHz)	Transverse mode (0,1) (kHz)	Transverse mode (0,4) (kHz)	Estimated h_2 (mm)	Estimated h_1 (mm)
0	65.80	28.45	191.90	7.300	0.000
30	66.50	29.30	192.45	7.297	0.106
60	66.65	29.35	192.75	7.264	0.126
90	66.70	29.35	193.50	7.200	0.137
120	66.75	29.35	193.85	7.168	0.145
150	66.80	29.30	194.05	7.140	0.150
180	66.85	29.30	194.15	7.127	0.156
210	66.85	29.30	194.35	7.112	0.158
240	66.95	29.35	194.60	7.089	0.172
270	67.00	29.30	194.90	7.054	0.178
300	67.00	29.30	195.05	7.043	0.179
330	67.05	29.30	195.40	7.012	0.187
360	67.10	29.30	195.70	6.984	0.195

Table 2: Thickness of Zn (h_2) and ZnO (h_1) layers estimated from resonance frequencies of Sample B.

Time (min)	Radial mode (kHz)	Transverse mode (0,1) (kHz)	Transverse mode (0,4) (kHz)	Estimated h_2 (mm)	Estimated h_1 (mm)
0	64.20	27.70	188.50	7.300	0.000
30	65.35	28.70	189.70	7.066	0.185
60	65.45	28.65	189.60	7.049	0.192
90	65.50	28.55	189.30	7.043	0.191
120	65.60	28.55	189.65	7.014	0.203
150	65.65	28.55	189.95	6.994	0.210
180	65.75	28.60	190.40	6.965	0.225
210	65.80	28.65	190.50	6.958	0.233
240	65.90	28.65	190.75	6.933	0.245
270	66.00	28.65	191.00	6.907	0.257
300	66.05	28.70	191.30	6.893	0.266
330	66.20	28.70	191.50	6.862	0.283
360	66.30	28.75	191.85	6.838	0.298

CONCLUSION AND FUTURE WORK

This article presents an extension of the graphical technique for crack and delamination quantification in beams to quantify the extension of corrosion in sacrificial zinc anodes with EMI measurements. The model shows good agreement in trend with that of the experiment for three modes of vibration. The accuracy of estimated thicknesses can be improved further by modelling the PZT transducer as an anisotropic material. The current study is conducted for sacrificial anodes in marine applications and hence only free edge boundary conditions are considered. The same can be extended to other applications in corrosion monitoring of RCC structures where simply supported and fixed edge boundary conditions come into play. This work enables low-cost implementations of cathodic protection systems using EMI with embedded platforms for in-situ health assessment in the field.

ACKNOWLEDGEMENTS

This work was supported by IMPRINT-2A by Department of Science & Technology, Government of India [SERB grant IMP/2018/001442] with additional financial support from Sanrachana Structural Strengthening Pvt. Ltd. [grant RD/0119-SSIMPQ2-001]. The authors thank Krishna Conchem Products Pvt. Ltd. for providing the zinc sacrificial anodes used for performing the experiments, Mr. Hrishikesh Belatikar at IIT Bombay for assistance with developing LabVIEW based data acquisition software, and Wadhwani Electronics Lab (WEL) at IIT Bombay for providing Agilent E4980A precision LCR meter.

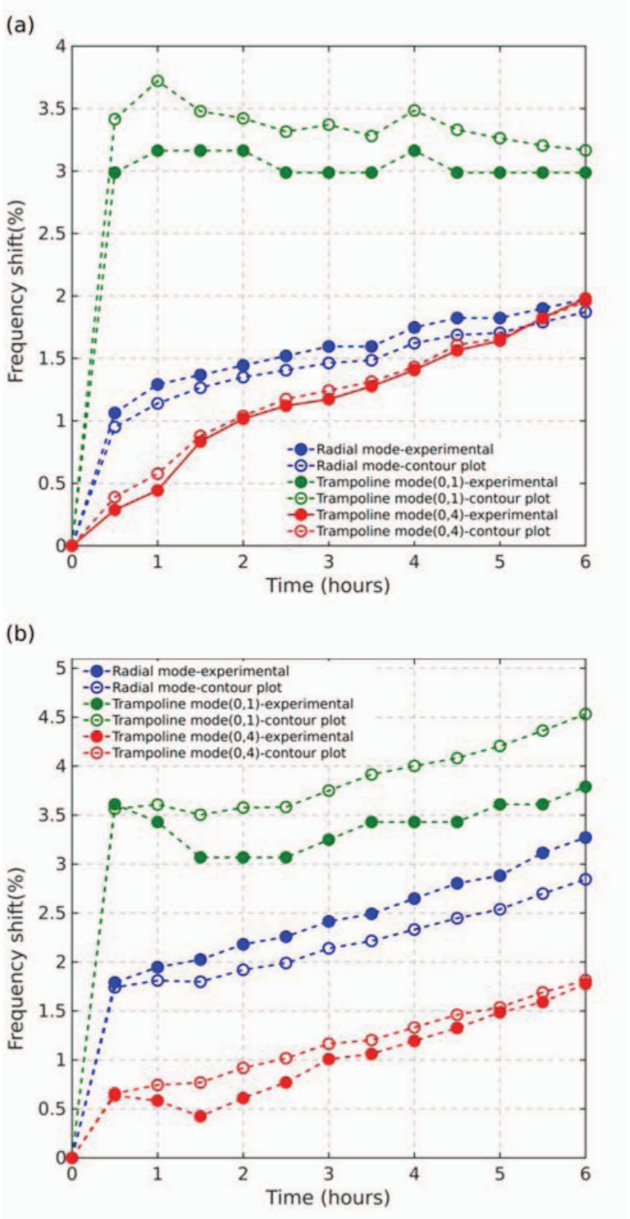

Figure 4: Frequency shift for fundamental radial mode and transverse vibration modes (0,1) & (0,4) for (a) sample A, and (b) sample B. The experimental results show excellent agreement with FEA validation of solutions obtained from the contour plot technique.

REFERENCES

[1] H. Sohn, C.R. Farrar, F.M. Hemez, “A Review of Structural Health Monitoring Literature 1996-2001,” *Proceedings of the 3rd World Conference on Structural Control*, Como, Italy: Los Alamos National Laboratory, USA, pp. 301, 2002.

[2] O. Salawu, “Detection of structural damage through changes in frequency: a review,” *Engineering Structures,* 19, pp. 718, 1997.

[3] J. Humar, A. Bagchi, H. Xu, “Performance of vibration-based techniques for the identification of structural damage,” *SAGE Journal of Structural Health Monitoring*, 5(3), pp.215-241, 2006.

[4] J. Ravi, S. Nidhan, N. Muthu, S. Maiti, “Analytical and experimental studies on detection of longitudinal, L and inverted T cracks in isotropic and bi-material beams based on changes in natural frequencies,” *Mechanical Systems and Signal Processing,* 101, pp. 67, 2018.

[5] Z. Zhang, K. Shankar, T. Ray, E. V. Morozov, M. Tahtali, “Vibration-based inverse algorithms for detection of delamination in composites,” *Composite Structures,* 102, pp. 226, 2013.

[6] B. Nandwana, S. Maiti, “Modelling of vibration of beam in presence of inclined edge or internal crack for its possible detection based on frequency measurements,” *Engineering Fracture Mechanics,* 58, pp. 193, 1997.

[7] J. Thalapil, S. Maiti, “Detection of longitudinal cracks in long and short beams using changes in natural frequencies,” *International Journal of Mechanical Sciences*, 83, pp. 38, 2014.

[8] R. Adams, P. Cawley, C. Pye, B. Stone, “A vibration technique for non-destructively assessing the integrity of structures,” *Journal of Mechanical Engineering Science*, 20, pp. 93, 1978.

[9] D. Tamhane, J. Thalapil, S. Banerjee, S. Tallur, “Smart cathodic protection system for real-time quantitative assessment of corrosion of sacricial anode based on Electro-Mechanical Impedance (EMI),” IEEE Access, vol. 9, pp. 12230-12240, 2021.

[10] V. Giurgiutiu, *Structural Health Monitoring with Piezoelectric Wafer Active Sensors (Second Edition)*, Academic Press, 2014.

[11] S.S. Rao, *Vibration of Continuous Systems (Second Edition)*, John Wiley & Sons, Inc., 2019.

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